V. *Observations on certain cases of Elliptic Polarization of Light by Reflexion.* By *the Rev.* Baden Powell, *M.A., F.R.S., F.G.S., F.R.A.S., Savilian Professor of Geometry in the University of Oxford.*

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Introduction.

THE peculiar character impressed on light, originally polarized in a plane inclined to that of incidence and reflected from a metallic surface, discovered by Sir D. Brewster*, and named by him elliptic polarization, has been since shown to coincide with what, from a different analogy, is termed elliptic polarization in the undulatory theory; and which is also exhibited by an interposed plate of mica, or by total internal reflexion, as in Fresnel's rhomb.

The most distinct experimental test of the existence of this property and a measure of its amount, is the well-known *dislocation* of the polarized rings, seen by a plate of calc-spar and a tourmaline, in light of this kind. And this, as in other similar cases, is represented theoretically by a formula for the intensity at any part of the plate, *in the case of the rhomb,* for *circular* polarization, as in Mr. Airy's tract on the undulatory theory (Art. 160.); and for *elliptic*, as in the same author's paper on quartz†. A general formula for the rings in light of all degrees of ellipticity, *not* restricted by the peculiar conditions of the *rhomb,* has not been hitherto published: but I am indebted to Mr. Airy for the communication of such a formula, which will be given in the sequel, as leading to some remarkable applications.

With immediate reference to the experiments of Sir D. Brewster *on thin films*‡, Professor Lloyd, in 1841, investigated on the principles of the undulatory theory, the general case of light previously polarized in any azimuth, and reflected from a thin plate, at any angle; and by generalizing the methods of M. Poisson, found expressions both for the intensity, the changes corresponding to different incidences, and the phase of vibration of the pencils reflected from the two surfaces, which in general differ in retardation and are polarized, one in the plane of reflexion and the other perpendicular to it: whence it follows that the resulting light should be in general *elliptically polarized*§.

Professor Lloyd's theory seems completely to explain the various phenomena ob-

* Philosophical Transactions, 1830.
† Cambridge Transactions, 1831.
‡ Philosophical Transactions, 1840.
§ See Reports of British Association, 1841, Sectional Proceedings, p. 26.

served by Sir D. BREWSTER, including those of Mr. AIRY* on the colours of thin plates in polarized light. But Professor LLOYD also infers the further application of the same principles to the case of elliptic polarization in the reflexion from polished metals, on the hypothesis that their superficial laminæ may be regarded as thin plates, or at least act in an analogous manner.

Before I was acquainted with Professor LLOYD's *theoretical* investigations, I had pursued an *experimental* examination of the phenomena of elliptic polarization in the reflexion from various surfaces, in the course of which I was led to some cases which seem to have a more special bearing upon theory, particularly in connexion with the views just referred to.

My observations were all conducted by the method of observing the modifications of the polarized rings under different conditions, both of surface and of incidence; and were directed to ascertaining both the *existence* and *amount* of ellipticity shown by the dislocation of those rings, as also to the peculiar character indicated by the *direction* in which the dislocation takes place; the protrusion of the alternate quadrants appearing in certain cases in one direction and in others in the opposite.

The observations are reducible to two classes:—1st, those designed to contribute to the inquiry *what substances* possess the property of elliptic polarization ?, by examining the light reflected from various bodies; of which I here notice only a few cases which appeared remarkable; 2ndly, observations on certain cases of *films* of several kinds, including those formed on metal by oxidation, or other action upon the metal itself, as well as by extraneous deposition. In these cases the ellipticity generally exists in different degrees, and with different characters as to direction, while in some instances it is destroyed or reduced to plane polarization.

I at first noticed these effects as produced in some cases of highly polished metal which had become tarnished by long exposure, and in which iridescent films had formed on the surface without destroying the polish. And again, in trying the effect of *heating* a metal plate, while observing the rings, I found the ellipticity disappear, but soon perceived that the effect was due to the coloured films formed on the surface, and remained when the plate was cold. I was thus led to institute more exact experiments of the same kind, in which the tints were formed in regular succession, as well as to examine steel in different stages of tempering.

From these cases I was naturally led to those of the films of metallic deposit produced by the galvanic process of NOBILI; specimens of which were kindly furnished me by Dr. DAUBENY. In these films it is well known the colours follow, at least generally, the orders of NEWTON's scale; the thickest film being deposited where the action is most intense, or where the surfaces in connexion with the two poles most nearly approach one another. In all these instances, then, I had a succession of films formed upon metal, in which the changes effected in the polarization could be traced in regular order.

* Cambridge Transactions, 1832.

The general result in all these cases, is that, *from any one tint to another, through each entire order of tints, the form of the rings in the reflected light undergoes certain regular changes, passing from a dislocation in one direction, to that in the opposite, through an intermediate point of no dislocation, or of plane polarization: and this, exhibiting a dark and a bright centred system alternately, as long as the orders of tints are preserved pure.*

Now these are precisely the changes in the form of the rings expressed by successive modifications of Mr. Airy's formula, corresponding to the increments in the retardation which belong to the periodical colours of the films.

In the instance of the metallic films it is a question whether in any case the existence of elliptic polarization be due to the action of the film simply, or whether the subjacent metallic surface have any share in producing it, while the film acts as a retarding plate, which would render the conception of the mode of action more complex.

These are points on which, perhaps, at present we cannot form a decisive opinion. But the fact that the ellipticity, to whatever cause it may be due, undergoes the changes just mentioned, affords an interesting comparison with theory, and may aid future advances towards a knowledge of the nature of the action which produces elliptic polarization in these cases.

Apparatus and Method of experimenting.

The arrangement of my apparatus was, essentially, as follows: the light was polarized by transmission through a Nicol-prism; this was attached to a small graduated arc, so that it could be adjusted to throw the polarized ray at any required incidence on the surface under examination; and could be turned about its own axis, so that the plane of polarization might be inclined to that of incidence.

After reflexion, the ray was received by an analysing apparatus, containing a plate of calc-spar and a tourmaline, capable of a corresponding adjustment to different inclinations, by which were exhibited the polarized rings in the several modifications they underwent in different cases. This part had also a motion about its own axis, measured by a graduated circle. It will hardly be necessary to state more details as to the construction of the apparatus, except perhaps to observe that an arrangement, by which the surface under examination could be slid horizontally under the polarizing apparatus, was necessary when the object was to examine the changes presented in passing from one part of some surfaces to another.

In many cases where the reflexion from parts of a surface of varying character was to be examined, I fitted to the eye-piece a lens of short focus, between the calc-spar and the reflecting surface, which enabled me to see with great distinctness the rings in light from very small portions of the surface, which could be isolated by covering the rest.

In all cases the analyser is supposed in the position to give the dislocated rings with the nearest approach to the dark cross with the plain metal; the circular systems alone being precisely described as dark or bright centred.

The direction of dislocation is distinguished by the quadrants in which the dark patches near the centre occur. The position of the line joining them, upon plain metal, is taken as the zero for comparison in other cases.

The observations were repeated at several different incidences, but for the purpose of the comparisons here in view, it suffices to give the results at one incidence, the relative appearances at others being similar. They are arranged in a tabular form in each case.

Observations.—Professor FORBES's *Mica.*

The original observations of Sir D. BREWSTER were confined to pure metals and a few metallic ores; in all which the ellipticity is insensible at incidences less than about 30°, and comes to a maximum at between 70° and 80°.

Besides these, as far as I am aware, the only instance is that announced by Professor FORBES* of elliptic polarization in the reflexion from mica when reduced to the particular state in which he used it for his experiments on heat.

On repeating the experiment I observed that it has its maximum at an incidence between 20° and 30°, and the direction of dislocation, 90°. But the films thus formed do not lose their crystalline structure or retarding property: it may therefore be doubtful how far the effect may be explicable in the ordinary way.

Decomposed Glass.

In some specimens of glass, whose surface is in a well-known peculiar state of decomposition, not only iridescent, but having a singularly metallic lustre, I found elliptic polarization, though none was perceptible in other specimens, however highly iridescent, which had not the metallic appearance. There are, however, anomalies with other metallic reflexion, for the maximum effect appears at *small* incidences (about 30° or 40°), and the direction of dislocation is 90°.

Minerals, &c.

Among a variety of metallic ores, I have found elliptic polarization produced only by a few having a decided metallic lustre. What proportion of metal may be necessary to give ellipticity, is an interesting question for future research. But one remarkable instance is that of *plumbago*, which gives a small though distinct ellipticity; while its composition is well known to be doubtful: but on the highest estimate it contains 95 of carbon to 5 of iron†.

In some ores exhibiting a natural iridescence, the results seem analogous to those about to be described in artificial surfaces of this kind.

* British Association, 1839, Sectional Proceedings, p. 6.

† THOMSON's Chemistry, i. 396. 6th Ed.

Oxidation.

The common effect of oxidation is to reduce the elliptic into plane polarization. This is the case with the oxidation produced on copper, &c. by a drop of dilute acid, and with the dull tarnished surface of most metals after long exposure.

A transparent film on the surface of polished metal may diminish or destroy the ellipticity, obviously from its refraction causing the rays to fall on the metal beneath at too small an angle.

Mercury, when pure, gives a large elliptic polarization; and even when the surface is coated with the film of oxide which so readily forms upon it, the ellipticity remains unaltered: the surface of the oxide, however, has a sort of metallic appearance.

Daguerreotype Plates.

With one of these plates on which a picture had been formed, at incidences of 60° and 70°, I could perceive no difference in the degree of ellipticity, or direction of dislocation, between those parts of the surface which remained bright, and those which had been most powerfully acted upon.

Tempered Steel.

With reference to the same objects I tried plates of steel in its ordinary state, and in two stages of tempering, viz. the yellow or straw-coloured, and the blue. The former is well known to be that formed at the lowest heat. The film or state of surface thus produced occasions changes both in the amount and direction of the dislocation.

Dr. THOMSON*, in describing the process of tempering, observes that it is a question whether the changes in colour be due to thin plates of an oxide simply, or whether there may not be different oxides produced in succession.

This last opinion agrees with the nature of my results, as well as with the absence of change of colour on altering the inclination. The results are as follows:—

Incidence.	Surface.	Polarization.	Centre.	Direction of dislocation.
70°	Steel, plain	Elliptic, large	....	0°
	Steel, tempered.			
	Yellow......	Elliptic, very small, or none	Dark...	0°
	Blue..........	Very small........	Dark...	90°

Coloured Films on Steel by Heat.

To examine the phenomena more precisely in their order of succession, I formed coloured films on plates of highly polished steel, about five inches square, by applying to the under side, at the centre, the flame of a spirit-lamp, when colours soon be-

* Chem. i. 386. 6th Ed.

gan to appear on the upper surface, and assumed the form of rings round the point of application of the heat.

Each tint appeared in succession, first at the centre, and thence extended itself. The first tints succeeded each other in the course of a few seconds; but the latter more slowly; until at length, after the heat had been continued for a considerable time, no further change took place.

The tints, proceeding in order from the outermost to the centre, may be described and grouped as follows:—

1. Reddish brown, crimson, deep purple, dark blue, light blue, faint yellow.
2. Faint red, light blue, pale reddish brown (at the centre).

The last two colours were very faint, and seemed covered with a sort of cloudy whiteness.

It is difficult to compare these tints with any part of NEWTON'S scale: but I have here grouped them in two divisions which seem clearly marked by a periodicity, while in the last two tints a peculiar character appears to prevail.

The supposition of a change in the degree of oxidation before referred to, seems to accord with the peculiar appearance of the later tints, while the periodicity of the former agrees with the supposition of thin plates up to that point.

In the observed phenomena of ellipticity there appear distinctions corresponding to the *regular* orders of tints; for in passing them in succession under the apparatus, corresponding changes in the amount and direction of dislocation occur, as far as the red of the 2nd set; while in the remaining part, the ellipticity is greatly diminished, and no change in direction takes place.

The figures 1, 2, &c. distinguish successive parts of each coloured space at which the changes occur.

Incidence.	Surface.	Polarization.	Centre.	Direction of dislocation.
70°	Steel, plain	Elliptic, large.	....	0°
	Films by heat.			
	1st Red	Elliptic, large	....	0°
	Blue...... 1	Plane............	Bright.	
	Blue...... 2	Elliptic, large	....	90°
	Yellow........	Plane............	Dark.	
	2nd Red	Elliptic, large	....	0°
	Blue, &c.......	Small............	Dark ..	0°

Coloured Films on Copper by Heat.

I formed colours in exactly the same way on plates of copper of the same size. The tints here do not appear exactly the same as those on steel, though there is some general correspondence: they follow this order, commencing from the outermost.

1st. Red, purple, pale yellow, yellow.

2nd. Red, green, dull brown.

The last two tints have a sombre appearance, different from the metallic lustre of the others: this (as in the last case) may accord with the supposition of a different degree of oxidation.

Here also I find similar changes in the ellipticity through the first tints, but none in the last two.

Incidence.	Surface.	Polarization.	Centre.	Direction of dislocation.
70°	Copper, plain ..	Elliptic, large.	....	0°
	Films by heat.			
	1st Red	Elliptic, large	....	0°
	Purple........	Plane...........	Bright.	
	Yellow.... 1	Elliptic, large	....	90°
	Yellow.... 2	Plane...........	Dark.	
	2nd Red	Elliptic, large	....	0°
	Green, &c.	Small...........	Dark ..	0°

Nobili's *Coloured Films*.

Besides furnishing me with several specimens of these films on steel plates, in which very brilliant tints were developed as far as the 3rd order, Dr. Daubeny kindly caused other plates to be prepared in his laboratory, in order to trace the order of the effects produced.

In forming one of these, the process was continued longer than before, and thus a 4th order of very faint colours succeeded. In another instance the process was continued still longer, and colours were formed which went through the same series as before, but at the 4th order they became extremely dusky and obscure; and the 5th order was barely visible from increasing opacity. They followed one another in succession towards the circumference, where they were at length crowded together, forming a narrow fringe, within which the central space was of a deep brown or black, of a dull, opake, appearance; and when this had occurred, the galvanic action being kept up for some time afterwards, no further change took place.

It can hardly be doubted that the increasing dulness in the later orders of tints, and final opacity of the film, are due to some change in the nature of the deposit which is superinduced at this stage of the process.

On bringing the coloured parts of the plate successively under the apparatus, the changes in the rings were clearly marked through the three orders of tints; though they followed in very close succession, commencing upon the edge of the film; and in the passage through the bright-centred system the variations of colour were striking, and at first somewhat perplexing.

The following general results were derived from repeated observations with different plates compared together.

Incidence.	Surface.			Polarization.	Centre.	Direction of dislocation.
70°	Steel, plain			Elliptic large	....	0°
	NOBILI's films.					
	First order	yellow	1	Elliptic, large	....	90
			2	Plane	Dark.	
		red	3	Elliptic, large	....	0
			4	Plane	Bright.	
	Second order	green	1	Elliptic, large	....	90
			2	Plane	Dark.	
		red	3	Elliptic, large	....	0
			4	Plane	Bright.	
	Third order	green	1	Elliptic, large	....	90
			2	Plane	Dark.	
		red	3	Elliptic, large	....	0
			4	Plane	Bright.	
	Fourth order	green		Elliptic, small	Dark.	0
		red		Elliptic, very small	Dark.	0
	Black			Elliptic, very small, or none	Dark.	0

Analytical Investigation.

Mr. AIRY's general formula is investigated as follows :—

In the usual notation of the undulatory theory, we suppose a vibration in a plane P, represented by

$$a \sin \frac{2\pi}{\lambda}(v t - x)$$

incident on a metallic surface, so that P is inclined 45° to the plane of reflexion R. Then (the vibration perpendicular to R being accelerated in phase by ϱ) the resolved parts are

$$\frac{a}{\sqrt{2}} \sin \frac{2\pi}{\lambda}(v t - x) \quad . \quad . \quad . \quad . \quad \text{in R.}$$

$$\frac{a}{\sqrt{2}} \sin \frac{2\pi}{\lambda}(v t - x + \varrho) \quad . \quad . \quad \text{perpendicular to R.}$$

Then for a plane Q in the crystallized plate inclined to R by an angle φ (omitting the constant coefficient) the vibration perpendicular to Q, giving the ordinary ray, O, will be

$$\sin\left(\frac{2\pi}{\lambda}(v t - x + \varrho)\right) \cos\varphi - \sin\frac{2\pi}{\lambda}(v t - x) \sin\varphi = \text{O},$$

and that in Q giving the extraordinary ray E will be

$$\sin\left(\frac{2\pi}{\lambda}(v t - x + \varrho)\right) \sin\varphi + \sin\frac{2\pi}{\lambda}(v t - x) \cos\varphi = \text{E}.$$

But on emergence from the crystallized plate the latter is accelerated by θ (dependent on the thickness traversed), or it becomes

$$\sin\varphi \sin\left(\frac{2\pi}{\lambda}(v t - x + \varrho) + \theta\right) + \cos\varphi \sin\left(\frac{2\pi}{\lambda}(v t - x) + \theta\right) = E'.$$

The parts remaining after analysation in a plane A perpendicular to P will then be,

$$O \cos(45 - \varphi) - E' \sin(45 - \varphi).$$

These upon expansion are at length reducible to the form

$$H \sin\frac{2\pi}{\lambda}(v t - x) \quad + K \cos\frac{2\pi}{\lambda}(v t - x),$$

whence, after reduction, we obtain

$$H^2 + K^2 = 1 - \sin^2 2\varphi \cos\varrho - \cos 2\varphi \sin\varrho \sin\theta - \cos^2 2\varphi \cos\varrho \cos\theta,$$

which expresses the intensity of light at any part of the rings, assigned by φ and θ; and is susceptible of variations according to the value of ϱ, or the changes in retardation which may arise from differences in the nature of the metallic films, as in the following Table:—

Values of ϱ.	Expression for the intensity: θ given.					Nature of the rings.
	For φ in general.	$\varphi = \begin{cases} 0 \\ 180°. \end{cases}$	$\begin{cases} 45°. \\ 225°. \end{cases}$	$\begin{cases} 90°. \\ 270°. \end{cases}$	$\begin{cases} 135°. \\ 315°. \end{cases}$	
0	$1 - \sin^2 2\varphi - \cos^2 2\varphi \cos\theta$	$1 - \cos\theta$ ..	0	$1 - \cos\theta$ ..	0	Dark-centred, circular.
$\frac{\pi}{2}$	$1 - \cos 2\varphi \sin\theta$	$1 - \sin\theta$ ..	1	$1 + \sin\theta$ ..	1	Dislocated (1).
π	$1 + \sin^2 2\varphi + \cos^2 2\varphi \cos\theta$	$1 + \cos\theta$ ..	2	$1 + \cos\theta$ ..	2	Bright-centred, circular.
$\frac{3\pi}{2}$	$1 + \cos 2\varphi \sin\theta$	$1 + \sin\theta$ ..	1	$1 - \sin\theta$ ..	1	Dislocated opposite way to (1).
2π	$1 - \sin^2 2\varphi - \cos^2 2\varphi \cos\theta$	$1 - \cos\theta$ ..	0	$1 - \cos\theta$ ..	0	Dark-centred, circular.
&c.	&c.	&c.	&c.	&c.	&c.	&c.

These changes will accord with those observed through one order of tints, if the increase of ϱ from 0 to 2π correspond to an increment of one wave length, or if $\varrho = \frac{2\pi k}{\lambda}$ and k be made successively $= 0, \frac{\lambda}{4}, \frac{\lambda}{2}$, &c.